

The Neutron Spin Structure Function from the Deuteron Data in the Resonance Region

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Abstract

Nuclear effects in the spin-dependent structure function g_1 of the deuteron are studied in the kinematics of future experiments at CEBAF, ($\nu \leq 3 \text{ GeV}$, $Q^2 \leq 2 \text{ GeV}^2$). The magnitude of nuclear effects is found to be significantly larger than the one occurring in deep inelastic scattering ($\nu \rightarrow \infty$, $Q^2 \rightarrow \infty$). A possibility to measure the neutron structure functions in the CEBAF experiments with deuterium is analysed. It is found that disregarding or improperly treating nuclear effects in the region of nucleon resonances would lead to the “extraction” of an unreliable function. A procedure aimed at correctly extracting the neutron structure function from the deuterium data is illustrated and conclusions about the experimental study of the Q^2 dependence of the Gerasimov-Drell-Hearn Sum Rule for the neutron are drawn.

I. Recently it has been proposed [1] at CEBAF to experimentally study the spin-dependent structure function (SF) of the neutron g_1^n , in a wide interval of energy ν ($0.2 - 3 \text{ GeV}$) and momentum transfers Q^2 ($0.15 - 2 \text{ GeV}^2$), using polarized deuterium and ^3He targets. These experiments will shed light on a number of quantum chromodynamics (QCD) sum rules and will help to establish a connection between results predicted by low energy theorems ($Q^2 \rightarrow 0$) and perturbative QCD ($Q^2 \gg m^2$, m being the nucleon mass).

Of particular interest is the Q^2 dependence of the Gerasimov-Drell-Hearn (GDH) Sum Rule for the neutron.

The GDH Sum Rule, which has been derived in the real photon limit ($Q^2 = 0$) by Gerasimov [2] and Drell and Hearn [3], reads as follows:

$$\frac{m^2}{8\pi^2\alpha} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} (\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)) = -\frac{\kappa^2}{4}, \quad (1)$$

where κ is the nucleon anomalous magnetic moment, ν_{th} is the threshold energy of the pion photo-production, $\sigma_{1/2(3/2)}$ is the absorption cross section for total helicity $1/2(3/2)$, α is the fine structure constant. The sum rule (1) can be generalized to the case of electron scattering by expressing the helicity cross sections, $\sigma_{1/2(3/2)}$, through the spin-dependent SF $g_1(x, Q^2)$ and $g_2(x, Q^2)$, obtaining [4]

$$I_{GDH}(Q^2) = \frac{2m^2}{Q^2} \int_0^{x_{max}} dx \left[g_1(x, Q^2) - \frac{4m^2 x^2}{Q^2} g_2(x, Q^2) \right]. \quad (2)$$

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Since under usual experimental conditions the second term in eq. (2) is small, one expects that

$$I_{GDH}(Q^2 = 0) = -\frac{\kappa^2}{4}, \quad (3)$$

i.e. a negative value for the first moment of the nucleon SF. Since the experimental data in the region of the deep inelastic scattering show a positive value for the first moment of proton SF [6, 7, 8], this implies that in the Q^2 -evolution of the GDH Sum Rule from the real photon limit to the deep inelastic region, the first moment of the proton SF must change its sign. The change in sign is expected to occur in the region of excitation of the nucleon resonances where neither perturbative QCD nor chiral theories are applicable. Indeed, the momentum transfer is already small enough to use the perturbative methods of QCD, whereas, at the same time, the chiral loops expansion does not work because of drastic changes of the helicity structure in the resonance region.

On the other hand side, the measurement of the neutron SF will essentially contribute to the analysis of the deep inelastic sum rules, such as the Bjorken sum rule (BSR) and Ellis-Jaffe sum rules (see, for instance ref. [5] and references therein). In the deep inelastic limit, $Q^2 \rightarrow \infty$, the BSR connects the difference of the first moments of the spin-dependent SF of the proton and the neutron with the axial constant of the neutron β -decay:

$$\int_0^1 dx (g_1^p(x) - g_1^n(x)) = \frac{1}{6} \left(\frac{g_A}{g_V} \right), \quad (4)$$

where $x = Q^2/2m\nu$ is the Bjorken scaling variable. The experimental check of eq. (4) has already begun [6, 7, 8] by a measurement of the SF of the proton and the neutron, using in the latter case nuclear targets, viz. deuterium and 3He . The experimental data from different groups give slightly different values for the BSR. However, theoretical efforts in computing the Q^2 -corrections recoincline the data with the theoretical prediction (4), leading to the conclusion that the BSR is experimentally confirmed with an accuracy of about two standard deviations in the measured interval of Q^2 [5]. Nevertheless, the investigation of the Q^2 -evolution of the BSR in a wide interval of Q^2 and the problem of its explanation within QCD remain of great interests. Beside the GDH and BSR sum rules, yielding integral characteristics of the SF, a detailed study of the resonance behavior of the nucleon SF $g_1(x, Q^2)$ is planned as well.

All above examples demonstrate that the measurement of the spin-dependent neutron SF at CEBAF will provide us with important new information about the nucleon structure and will help to test a number of theoretical models and methods.

Keeping in mind the lessons we have learned from the EMC-effect, one might expect that nuclear corrections could play an important role in estimating the neutron SF from the combined nuclear and proton data [9, 10]. In the region of finite $Q^2 \sim m^2$ and $\nu \sim m$, nuclear corrections are much more important than in the deep inelastic limit [11]. In this paper the role of nuclear structure effects in electron-deuteron scattering in the resonance region will be discussed, paying special attention to the procedure of the extraction of the neutron SF from the deuteron data in the kinematics of future experiments at CEBAF.

II. The nucleon contribution to the deuteron structure functions is usually calculated by weighting the amplitude of electron scattering on the nucleon with the wave function of nucleon in the deuteron (for recent developments see e.g. [12, 13, 14, 15] and references therein). For the spin-dependent SF the most important effects are the Fermi motion and the depolarizing effect of the D-wave. Additional effects, such as off-mass-shell effects or nucleon deformation, are found to be small [16, 17]. For finite values of Q^2 and ν , the deuteron SF $g_1^D(x, Q^2)$ reads as follows

$$g_1^D(x, Q^2) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{m\nu}{kq} g_1^N(x^*, Q^2) \left(1 + \frac{\xi(x, Q^2)k_3}{m}\right) (\Psi_D^{M+}(\mathbf{k}) S_z \Psi_D^M(\mathbf{k}))_{M=1} \quad (5)$$

$$= \int_{y_{\min}(x, Q^2)}^{y_{\max}(x, Q^2)} \frac{dy}{y} g_1^N(x/y, Q^2) \vec{f}_D(y, \xi(x, Q^2)), \quad (6)$$

where $g_1^N = (g_1^p + g_1^n)/2$ is the isoscalar nucleon SF and $\Psi_D^M(\mathbf{k})$ the deuteron wave function with spin projection M . In the rest-frame of the deuteron, with \mathbf{q} opposite the z-axis, kinematical variables are defined as:

$$kq = \nu(k_0 + \xi(x, Q^2)k_3), \quad k_0 = m + \epsilon_D - \mathbf{k}^2/2m, \quad (7)$$

$$\xi \equiv q_3/\nu = |\mathbf{q}|/\nu = \sqrt{1 + 4m^2x^2/Q^2}, \quad Q^2 \equiv -q^2, \quad x^* = Q^2/2kq, \quad (8)$$

where $\epsilon_D = -2.2246 \text{ MeV}$ is the deuteron binding energy. The limits $y_{\min(\max)}(x, Q^2)$ are defined to provide an integration over the physical region of momentum in (5) and to take into account the pion production threshold in the virtual photon-virtual nucleon scattering². Since both $y_{\min}(x, Q^2)$ and $y_{\max}(x, Q^2)$ are solutions of a transcendent equation, explicit expressions for them cannot be given. However, in our numeric calculations they are accurately taken into account.

Eqs. (5)-(6) have the correct limit in the deep inelastic kinematics ($Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$). In this case: $\xi(x, Q^2) \rightarrow 1$, $y_{\min} \rightarrow x$, $y_{\max} \rightarrow M_D/m$, and the usual convolution formula for the deuteron SF [12, 15] is recovered:

$$g_1^D(x, Q^2) = \int_x^{M_D/m} \frac{dy}{y} g_1^N(x/y, Q^2) \vec{f}_D(y). \quad (9)$$

Equation (9) defines spin-dependent “effective distribution of the nucleons”, \vec{f}_D , which describes the bulk of the nuclear effects in g_1^D . The main features of the distribution function, $\vec{f}_D(y)$, are a sharp maximum at $y = 1 + \epsilon_D/2m \approx 0.999$ and a normalization given by $(1 - 3/2P_D)$ (P_D being the weight of the D-wave in the deuteron). As a result in the region of medium values of $x \sim 0.2 - 0.6$ the deuteron SF $g_1^D(x)$ is slightly suppressed by a depolarization factor, $(1 - 3/2P_D) \times g_1^N(x)$, compared to the free nucleon SF. However,

²For x not too close to the limit of single-nucleon kinematics, $x \rightarrow 1$, the quasi elastic contribution can be disregarded

the magnitude of this suppression is small ($\sim 1\%$) and this is why it is phenomenologically acceptable to extract the neutron SF from the deuteron and proton data by making use of the following approximate formula:

$$g_1^D(x, Q^2) \approx \left(1 - \frac{3}{2}P_D\right) (g_1^n(x, Q^2) + g_1^p(x, Q^2))/2. \quad (10)$$

In addition, when integrated over x , eqs. (9) and (10) give exactly the same result ($\Gamma = \int dx g_1(x)$), i.e.

$$\Gamma_D(Q^2) = \left(1 - \frac{3}{2}P_D\right) (\Gamma_n(Q^2) + \Gamma_p(Q^2))/2, \quad (11)$$

which allows to define *exactly* the integral of the neutron SF Γ_n from the deuteron and proton integrals, without solving (9).

Eqs. (5)-(6) at finite values of Q^2 and ν are more sophisticated than the corresponding equations in the deep inelastic limit. In particular, they do not represent a “convolution formula” in the usual sense, since the effective distribution function \vec{f}_D and the integration limits are also functions of x . This circumstance immediately leads to the conclusion that, in principle, when integrals of the SF are considered, the effective distribution can not be integrated out to get the factor similar to $(1 - 3/2P_D)$ in (11). Another interesting feature of formulae (5)-(6) is the Q^2 -dependence of \vec{f}_D and $y_{min,(max)}(x, Q^2)$. If we again limit ourselves to the discussion of the integrals of SF, one concludes that the Q^2 -dependence of such an integral is governed by both the QCD-evolution of the nucleon SF and the kinematical Q^2 -dependence of the effective distribution of nucleons.

Thus, we have established that in the non-asymptotic regime, equation (11), in principle, does not hold. Furthermore, it is not clear whether an equation similar to (10) could be applied in this region. Indeed, we are discussing the kinematical conditions pertaining to nucleon resonances, where the “elementary” nucleon SF explicitly exhibits Breit-Wigner resonance structures corresponding to the excitations of the nucleon by the photon and one expects that the Fermi motion and binding of nucleons will result in a shift and a smearing of the resonance structures. However, one can hope that actual effects will be quantitatively small so that eq. similar to (11) and (10) could phenomenologically still be valid.

III. In our numerical estimates we use a reliable parametrization of the proton and neutron SF given by Burkert [18], which takes into account several nucleon excitations and provides a reasonable description of the available nucleon data in the resonance region. Using the Bonn potential model for the deuteron wave function [19], we carry out a realistic calculation of the deuteron SF, $g_1^D(x, Q^2)$ in the region of nucleon resonances.

In Fig. 1 the results of the calculation of the deuteron SF, $g_1^D(x, Q^2)$ at $Q^2 = 0.1$ GeV and 1.0 GeV, are compared with the input of the calculation, i.e. the isoscalar nucleon SF, $g_1^N(x, Q^2)$. It can be seen that the role of nuclear effects in the resonance region is much larger (up to $\sim 50\%$ in the maxima of the resonances), than in the deep inelastic regime ($\sim 7 - 9\%$, depending upon the models [12, 13, 14, 15], with resulting $\sim 6 - 7\%$ from the depolarization factor $(1 - 3/2P_D)$ and $\sim 1 - 2\%$ from the binding effects and Fermi

motion). Such a drastic effect is a consequence of the presence of the narrow resonance peaks in the nucleon SF.

Fig. 2 shows the results of extraction of the neutron SF from the deuteron and proton data by using the approximate formula (10) which, we believe to give an upper limit of the possible errors in this extraction. To emphasize the role of nuclear effects in the region of finite Q^2 , the extracted neutron SF is compared with the original (input in the calculation) parametrization of the neutron SF. The use of the approximate formula (10) appears to be in some regions completely unreliable. This can be easily understood as follows: the proton and neutron SF have similar behavior in the resonance region, in that the positions of the nucleon resonances are the same for both of them, whereas the resonances in the resulting deuteron SF are smeared and shifted, compared to the isoscalar SF. Therefore, subtraction the proton SF from the deuteron one, in the maximum of the former, can result in a minimum for the neutron SF, instead of a maximum. The conclusion of our analysis is that nuclear effects in the resonance region are very specific and the approximate formula (10) does not work even for crude extraction of the neutron SF. Obviously, another method of extracting the neutron SF should be used.

In ref. [20] a rigorous method of solving eq. (9) for the unknown neutron SF has been proposed and applied in the deep inelastic region. It has been shown that this method, which works for both spin-independent and spin-dependent SF, in principle allows one to extract the neutron SF exactly, requiring only the analyticity of SF. It can also be applied by a minor modification to the extraction of the SF at finite Q^2 , which is our present aim.

The basic idea is to replace the integral equation (6) by a set of linear algebraic equations, $K\mathcal{G}_N = \mathcal{G}_D$, where K is a square matrix (depending upon the deuteron model), \mathcal{G}_D is a vector of the experimentally known deuteron SF and \mathcal{G}_N is a vector of unknown solution. Changing the integration variable in (6), $\tau = x/y$, we get

$$g_1^D(x, Q^2) = \int_{\tau_{min}(x, Q^2)}^{\tau_{max}(x, Q^2)} d\tau g_1^N(\tau, Q^2) \frac{1}{\tau} \vec{f}_D(x/\tau, \xi(x, Q^2)), \quad (12)$$

where $\tau_{min}(x, Q^2) = x/y_{max}(x, Q^2)$, $\tau_{max}(x, Q^2) = x_{max}(Q^2)/y_{min}(x, Q^2)$ and $x_{max}(Q^2)$ is defined by the pion production threshold in virtual photon-nucleon scattering. Let us assume that the deuteron SF has been measured experimentally in the interval (x_1, x_2) and a reasonable parametrization for the SF is found in this interval. Then, dividing both intervals (x_1, x_2) and (τ_{min}, τ_{max}) into N small parts, one may write:

$$g_1^D(x_i, Q^2) \approx \sum_{j=1}^N g_1^N(\tilde{\tau}_j, Q^2) \int_{\tau_j}^{\tau_{j+1}} \frac{1}{\tau} \vec{f}_D(x_i/\tau, Q^2) d\tau, \quad i = 1 \dots N, \quad (13)$$

where $\tilde{\tau}_j = \tau_{min} + h(j - 1/2)$ and $h = (\tau_{max} - \tau_{min})/N$. Equation (13) is already explicitly of the form $\mathcal{G}_D = K\mathcal{G}_N$, therefore the usual linear algebra methods can be applied to solve it.

Note that the range of variation of τ is larger than the one for x . Therefore, in principle, the SF of the deuteron experimentally known in the interval (x_1, x_2) contains

information about neutron SF in wider interval (for example, in deep inelastic regime $\tau_{min} \approx x/2$ and $\tau_{max} = 1$). However extracting information beyond the interval $\tilde{\tau}_{min} = x_1$ to $\tilde{\tau}_{max} = x_2$ is almost impossible in view of the structure of the kernel of eq. (12) and the kinematical condition of planned experimental data [20]. We have to redefine the kernel of eq. (12) to incorporate new limits of integration $\tilde{\tau}_{min} = x_1$ and $\tilde{\tau}_{max} = x_2$ [20].

The procedure of solving eqs. (6) in the kinematical region of finite Q^2 and ν will be presented elsewhere in details; here we only stress that the method works with a good accuracy. To check it, we calculated the deuteron SF by formula (6) with the nucleon SF $g_1^N(x, Q^2)$ from ref. [18] and the deuteron wave function of the Bonn potential [19]. Then the obtained $g_1^D(x, Q^2)$ has been used as “experimental” data to calculate the vector \mathcal{G}_D in (13); the matrix K has been calculated by using the same deuteron wave function. Equation (13) has been solved numerically for various “experimental” situations (changing the “measured” interval (x_1, x_2) , for different Q^2 , etc.). The obtained solution, i.e. the extracted neutron SF, has been compared point by point with the input to the calculation of g_1^D . We found that method is stable and allows one to unfold the neutron SF with errors not larger than 10^{-4} , which is much smaller than the expected experimental errors (note, that in the present paper we discuss only the spin dependent SF, nevertheless all results and conclusions are valid for unpolarized structure function $F_2(x, Q^2)$ as well.)

IV. In this section we discuss the role of nuclear corrections in the analysis of the integrals of the SF, such as the GDH Sum Rule.

A very important observation has been made in the deep inelastic limit, namely the *exact* formula (9) and the *approximate* formula (10) give the same result for the integral of the neutron structure function, $g_1^n(x, Q^2 \gg m^2)$ (see eq. (11)). The applicability of the approximate formula in the deep inelastic region is based on the conservation of the norm of the distribution $\vec{f}(y)$ by the convolution formula (9). This circumstance can not be immediately extended to the case of the resonance region, since: (i) the convolution is broken in eq. (6) and (ii) the normalization of the function $\vec{f}(y, x, Q^2)$ is different from one of $\vec{f}(y)$. The integral of the distribution $\vec{f}(y, x, Q^2)$ represents the “effective number” of nucleons “seen” by the virtual photon in the process when the virtual photon is absorbed by the nucleon and at least one pion is produced in the final state (it is less than 1 at low Q^2 and $x \rightarrow x_{max}$).

However, surprisingly enough, the use of the formula (11) in the resonance region gives results numerically very close to the integration of the exact equation (6). This is a consequence of the smallness of the effects breaking the convolution in eq. (6). These effects can be accounted for by a new equation:

$$\Gamma_D(Q^2) = \left(1 - \frac{3}{2}P_D\right) N_{eff}(Q^2)(\Gamma_n(Q^2) + \Gamma_p(Q^2))/2 \quad (14)$$

Eq. (14) and the integral of eq. (6) represent the definition of the “effective number” $N_{eff}(Q^2)$; the latter depends upon the form of the nucleon SF g_1^N , and, since this is expected to strongly oscillates (see Fig. 1), even the sign of the correction can vary. For instance we obtain using the SF from [18],

$$N_{eff}(Q^2 = 0.1 \text{ GeV}^2) = 1.02, \quad N_{eff}(Q^2 = 1.0 \text{ GeV}^2) = 0.997, \quad (15)$$

i.e. a rather small effect (+2% and -0.3% correspondly). Therefore eq. (14) appears to be a reliable one for estimating the integrals of SF: setting $n_{eff}(Q^2) = 1$ does not lead to errors larger than 3% for $Q^2 = 0.1 - 2.0 \text{ GeV}^2$.

V. In conclusion, we have shown that the effects of nuclear structure in the extraction of the neutron SF in the resonance region are much more important than in the deep inelastic scattering. We have explained how the correct neutron SF can be firmly extracted from the combined deuteron and proton data. At the same time, we have found that the integrals of the SF, such as the GDH Sum Rule, can be estimated with accuracy better than 3% by the simple formula (11) which is also valid in deep inelastic region.

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Figure captions:

Figure 1. The spin dependent structure functions $g_1(x, Q^2)$ for two values of Q^2 . The deuteron SF (solid line) is compared with the isoscalar nucleon SF (dotted line) used as input in the calculation of eq. (6).

Figure 2. The neutron SF (solid line) extracted by the approximate formula (eq. (10)) compared with the original parametrization (dashed line) used in the convolution formula (6).

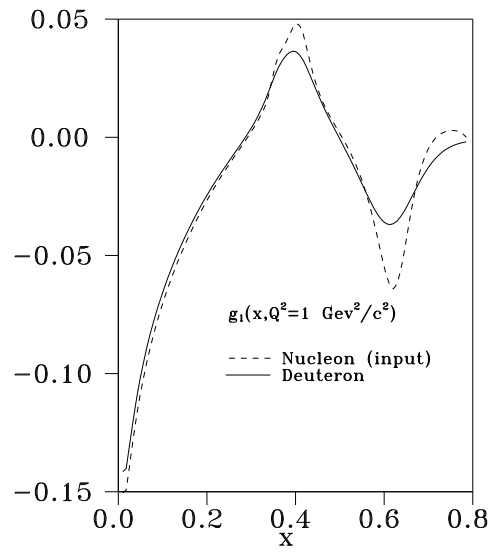
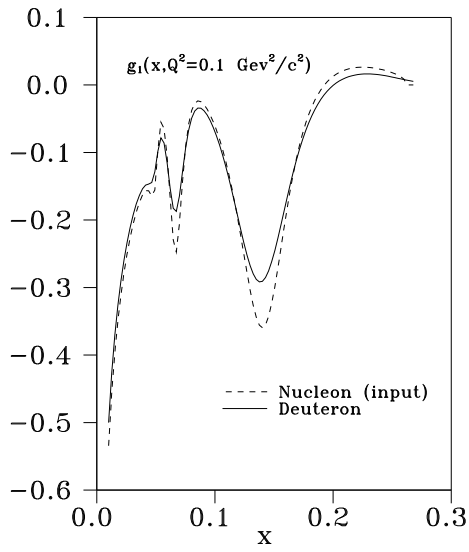


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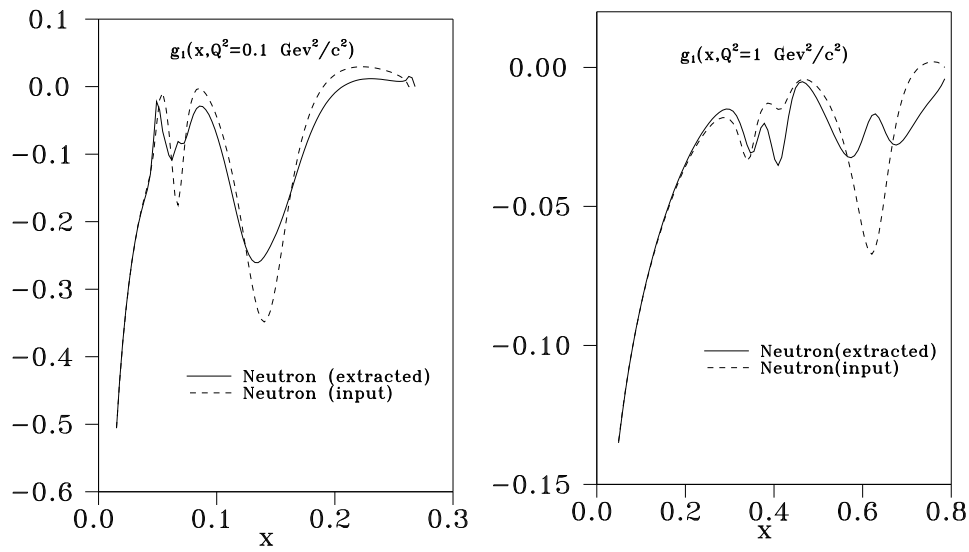


Figure 2: C. Ciofi degli Atti et al, The Neutron Spin Structure Function from...